# “On the expansion methods for numerical approximations to high-dimensional parabolic PDEs with non-constant coefficients”

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As a Sylff fellow and a Doctoral candidate at the Business School of the University of New South Wales (UNSW) Australia I had a chance to participate in the Sylff Research Abroad program with the aim to make a research visit to overseas research institution. This option appealed to me very much, since I was interested in getting research experience abroad and, more importantly, I wished to enrich my PhD thesis by adding a highly quantitative component into it. In particular, I aimed to add a rigorous analysis of the numerical methods employed in the financial modelling process.

The PhD projects, which I have been working on in UNSW, deal with pricing of financial derivatives, namely contracts, whose value depends on the future value of other assets, called underlying. Derivatives play an important role in the risk management process undertaken by various market participants, who want to get a certain level of financial protection against some unpredictable changes in the economy. An accurate mathematical modelling of all risks associated with a contingent claim is a crucial part of its valuation process. To get better understanding of these mathematical techniques was the prime goal of my research plan. That is why the institute, which I was looking for to undertake the research with SRA at, should be focused on the numerical modelling of the financial assets. I was invited to work for five months in the Mathematical and Computational Finance group at the Mathematical Institute at the University of Oxford, United Kingdom. Being one of the best research groups in the area of quantitative and mathematical finance, it provided me with the opportunity to collaborate with experts in the field, to sharpen my programming skills and to expand knowledge in numerical methods.

The goal of the research project which I was working on in the Mathematical Institute was to analyse the errors that arise from approximations of the solutions to mathematical models used for pricing of financial contingent claims written on many underlying assets and/or depending on many risk factors. The example of these contracts can be a basket option written on several stocks or indices. The main challenge with valuation of this kind of contracts is computational complexity associated with the process of finding a solution to the equation, satisfied by the price of the derivative. Due to high-dimensionality of the problem and complexity of the model it is puzzling to find an exact solution to the pricing equation. In this situation one has to resolve to some computational methods, which allow to get an approximation to the exact solution with a certain level of accuracy. Usually there is a trade-off between the precision of the numerical solution and the computational time required to obtain this solution.

There is a broad scope of approaches suggested in the literature on the numerical approximation of the prices of multi-asset options. However, the traditional methods based on the discrete approximation of the continuous solution, while being accurate for lowdimensional problems, suffer from the “curse of dimensionality”. Specifically, the computational time grows exponentially with the number of dimensions (e.g. number of assets) of the problem. It makes these methods being intractable for using in pricing of multi-asset derivatives.

The approach analysed in the research project flows from the observation that there is usually a low number of factors influencing the dynamics of the financial markets. More precisely, the number of principal components that describe the dominant part of the changes of values of financial assets is lower or equal to the number of assets, which the contingent claim is written on. This idea suggests that one could select the principal components and represent the price of the derivative in terms of these components only. It allows to reduce the dimensionality of the problem without significant reduction in accuracy. Our theoretical analysis suggests that the computational time grows linearly with the number of dimensions, which result in significant reduction of the computational complexity and increases the efficiency of the algorithm.

We develop a rigorous theoretical framework for the errors associated with the method: choose the mathematical models having a high relevancy to the financial engineering and derive the upper bounds on the approximation errors. In order to compliment the theoretical component of the research we perform a series of computational experiments. We observe a sound agreement between the theoretical findings and the results of the numerical simulations. This allows to establish a functional form of dependence between the approximation errors and the parameters of the mathematical models used to describe the dynamics of the underlying assets. We find a strong dependency between the numerical accuracy of the computational method and the specification of the mathematical model used to represent the dynamics of the value of the underlying asset. This suggests the directions of our further work. We are specifically interested in analysing of modelling framework with free-boundary problems associated with early exercise options and processes with non-constant parameters.

The results of the research project will become an integral part of my doctoral dissertation. The project compliments the other components of my thesis by adding a highly quantitative analysis of the methods used for pricing of financial derivatives under multiple risk factors and/or written on several underlying assets. I plan to summarise the main findings in the form of a research paper to be submitted for publication in an academic journal and to make a presentation at the local conference in the end of the year.

By making the results publically available, I hope to expand the existing literature on numerical methods employed in valuation of complex contingent claims. Benefits arising from the implementation of the results of the research in practice are as follows: the numerical methods which we analyse will help to improve the accuracy and time efficiency of risk computations in financial institutions. When used on practice these methods should allow assets to be fairly priced and to be easier to employ for the benefit of the general public. This should help to improve the stability and the efficiency of the market of exotic financial derivatives and have an overall positive contribution to the economy.